

**A STUDY OF DYNAMIC LOADING
ON SOIL**

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by

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FOREWARD

Interest in the field of vibration phenomena developed as a result of the authors observation of a testing program being conducted in California under the direction of L.W. Nijboer and C. Van Der Poel. The program involved testing of highway pavements to evaluate their trafficability. Comments were made at that time concerning the relationship between soil properties and performance under vibration.

Encouragement from Professor E.F. Kilcawley lead to this investigation, which represents an attempt to study some of the basic relationships between dynamic loading and soil properties.

The author wishes to express appreciation to Associate Professor Paul Lieber of the Geology Department and Associate Professor L.T. Assini of the Mechanics Department for assistance in evaluating the many problems which arose at the outset of the investigation.

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ABSTRACT

This study represents an attempt to develop a method for the laboratory investigation of dynamic loading of soils. The relationship between static properties of soils and the wave length and propagation velocity can be obtained through a controlled application of dynamic loading.

Soil is loaded by a plunger impinging on a spring loaded bearing plate to produce a sinusoidal loading force. The bearing plate transfers the load to a tube of soil four inches in diameter and ten feet in length.

Holes in the tube at one foot intervals provide access to the soil for a pressure transducer. The pressure transducer utilizes a flat diaphragm on which SR-4 Strain Gages are mounted. An unbalanced Wheatstone bridge is used to provide a voltage which is amplified and recorded on an oscilloscope.

The wave form is recorded for each access position along the tube. A marking pulse is superimposed on the oscilloscope by Z axis modulation. The pulse is produced by a set of contacts which close once for each cycle of loading at the same point in the cycle. The wave form along the length of the tube is then obtained from the position of the marking pulse on the wave at each access position. The wave length is obtained by measuring the distance between peaks on the wave form.

Wave lengths varied between fifteen and forty inches for forcing frequencies of 14.5, 21.75, and 29 cycles per second.

The propagation velocity is obtained by multiplying the frequency by the wave length. The velocities ranged from 40 to 60 feet per second.

The velocities and wave lengths differ considerably from values obtained by other investigators. This is attributed to the use of a soil tube. The nature of the waves is unknown.

The wave length of the recorded waves became shorter as the wave moved down the tube. This is considered to be caused by either damping or a change in density due to compaction of the soil in the tube under load.

It is believed that the relationship between soil properties and dynamic loading in a tube can be determined in a qualitative sense from such a procedure.

PART I
INTRODUCTION

A. Historical Review

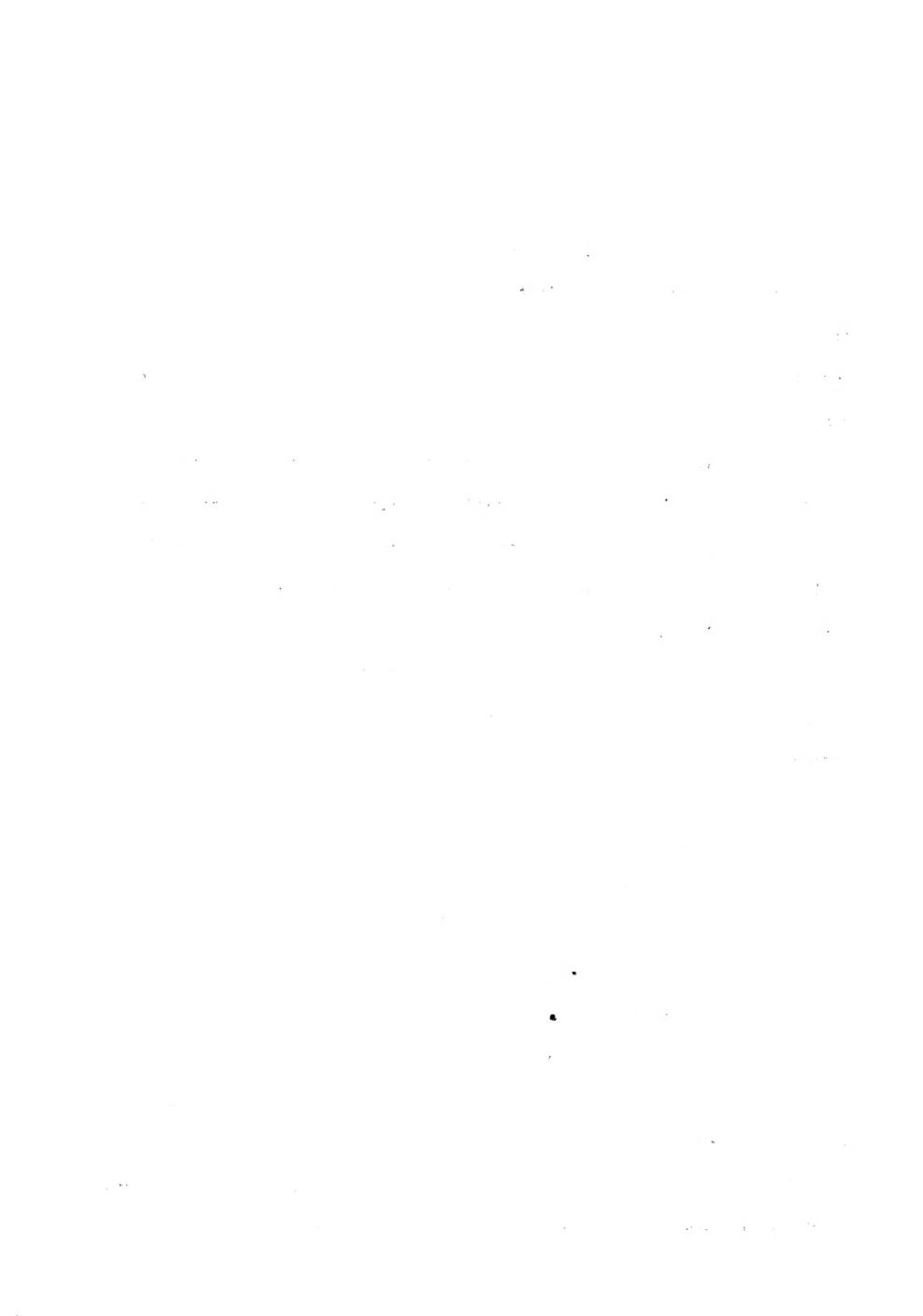
The importance of the effect of dynamic forces in machine foundations and structures has long been recognized. The effect of such forces in soils has been studied only recently, especially concerning performance in engineering works.

Early studies of vibrations in soils were performed in Europe for the purpose of exploring foundation conditions.^{7,11} These studies showed that characteristic resonant frequencies can be determined for soils which depend upon the site conditions and the type of soil. Application of some of the principals thus developed was made to highway pavements in Germany^{8,16} in 1934. Evaluations were made of the stiffness and trafficability of pavements. The relationships pertaining to density were utilized in dam construction in 1935.

These efforts precipitated a great deal of work concerned with resonance phenomena^{10,17,18} particularly as resonance might be developed as a method for obtaining compaction in soils. In 1939 Steuerman discussed a "New Soil Compaction Device"²¹ utilizing dynamic principals.

During this same period a new technique for obtaining compaction in submerged soils was developed in Russia.²⁰ It is a method for the introduction of suitable liquids into a granular mass under simultaneous vibration of the mass and is now called "Vibroflotation". Many investigators have since studied this process.^{5,6}

With these studies of dynamic forces on soils, analytical concepts of the phenomena were proposed. Soil has been historically studied



as an elastic solid and, therefore, early explanations of behavior under dynamic loading utilized the elastic properties to predict the forces in the soil. Ehlers in 1942⁴ proposed modeling soils as a spring of a vibrating system. Many variations of such an approach have been proposed in an attempt to facilitate the prediction of behavior of soils subjected to dynamic forces. Quinlan¹⁶ and Sung²⁶ derived equations for the vibrating soil system, neglecting damping in order to predict resonance. Slade¹⁹ imposed a transition zone between the oscillator and the area of the soil to be analyzed, and suggested that any mathematical model applied to soils can have only very limited application.

Winterkorn²⁵, in an attempt to broaden the concept of modeling soil, proposed the consideration of soil as a macrometric liquid as opposed to the classic concept of soil as an elastic solid.

In order to account for the way in which size and shape of the contact area, and the effect of the soil modulus increasing with depth affect the result, Pauw¹⁵ modeled the soil as a truncated pyramid of "soil springs".

Out of all of this work on modeling, the idea that soil could be considered to be a spring was found to be inadequate. Lorenz¹² states that the assumption that soil acts as a linear spring without mass, does not lead to satisfactory results. The spring used in a mathematical model does not obey "Hooke's Law" in deformation. Ho developed graphical methods to modify this assumption.

The work done on classifying properties of soils through dynamic loading has been, on a broad scale, associated with particular problems. Tschobotarioff²³ has compiled data on the natural frequency of soils classified into peats, plastic clays, sands, gravels, and rock. Peats and plastic clays have low natural frequencies while sands have relatively high natural frequencies. Nyboer and Van Der Poel¹⁴ presented data indicating low velocities for peats and clays and high velocities for sands. Bernhard and Finelli², in discussing a broad program of study of dynamic properties in soils, confirm the data given by Nyboer and Van Der Poel. Little information is presently available concerning the effect of moisture content, grain size distribution, or fine grain soils on dynamic properties.

B. Statement of the Problem

A great deal of study has been devoted to the determination of resonant frequencies in soils and in particular the process of compaction utilizing vibratory methods. In general, the equipment used in these studies has been heavy and most of the work conducted in the field.

Verification of theories and results under controlled conditions has been left largely to chance, particularly, concerning the various phenomena associated with wave forces and the relationships between the common engineering properties of soils of a static nature. Most engineering structures are built by specifying properties of soils such as density, grain size distribution, plasticity index, and moisture content.

This study, therefore, is an attempt to develop equipment in the form of a small, simple testing apparatus for laboratory use which will facilitate investigation of wave forms, wave length and velocity of compression waves produced by dynamic loading.

Such an apparatus, inevitably, must be the result of compromises and will have limitations based on the validity and application of assumptions which are necessary to provide a basis of design.

An attempt is made in the design of the loading system to obtain a sinusoidal pulse in a horizontal tube of sand in order to permit an evaluation of wave form and wave length. The nature of the forces within the soil, as a result of such a loading, are affected by the physical limitations of the containing tube. The effect of reflection of waves from the inside surface of the tube are assumed to be negligible. The degree to which the tube eliminates wave forms other than the compression wave will have a marked effect on the reliability of data obtained.

A device for recording impulses in soil cannot possess all of the desirable qualities necessary to accurately record the nature of the forces involved. A compromise was necessary from the standpoint of practical convenience and physical limitations. The true nature of the forces which are recorded may differ considerably from the assumed condition.

While many limitations are built into this system, they do not necessarily affect the qualitative information which this apparatus

was built to obtain. If the dynamic properties of soils can be classified in a relative order, the application of such information to field problems can still prove to be of value. It is felt that basic information concerning wave form is necessary in the field of dynamic loading of soils.

PART II

THEORY

A. Analysis of the Vibrating System

1. Forced Vibration. The application of dynamic loading to soil systems has been classically modeled by considering the soil as an elastic material in a vibrating system. Such a system may be further simplified by considering a vibrating system possessing one degree of freedom with a linear spring constant and viscous damping.

A system of this type is shown in Fig. (1) having the differential equation of motion

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = F \sin \omega t \quad (1)$$

where m = vibrating mass

c = damping constant

k = spring constant

ω = angular velocity

$F \sin \omega t$ = exciting force

t = time

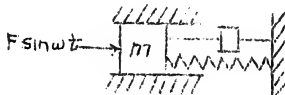


FIG. 1 - MODEL OF THE VIBRATING SYSTEM

The vibration represented by this equation may be separated into two types of motion, a free vibration with damping and a forced vibration. The forced vibration may then be indicated as

$$\frac{d^2 x}{dt^2} + 2p \frac{dx}{dt} + q^2 x = \frac{F}{m} \sin \omega t \quad (2)$$

where $q^2 = \frac{k}{m}$ and $2p = \frac{c}{m}$.

The complete solution for equation (2) requires a solution for the complementary function

$$\frac{d^2 x}{dt^2} + 2p \frac{dx}{dt} + q^2 x = 0 \quad (3)$$

The function $x = D e^{st}$ is a general function of t in which each successive derivative of x with respect to t is equal to the original function times a constant, in which D and s are unknown constants to be determined from initial conditions.²⁶

Therefore

$$\frac{dx}{dt} = D s e^{st}$$

$$\frac{d^2 x}{dt^2} = D s^2 e^{st}$$

Substituting these values in equation (3) gives

$$(s^2 + 2ps + q^2) C e^{st} = 0 \quad (4)$$

If the conditions of motion are such that

$$s^2 + 2ps + q^2 = 0$$

equation (3) is satisfied. The values of s that satisfy this condition are

$$s_1 = -p + \sqrt{p^2 - q^2} \quad \& \quad s_2 = -p - \sqrt{p^2 - q^2} \quad (5)$$

Thus the general solution of equation (3) is

$$x = C_1 e^{s_1 t} + C_2 e^{s_2 t} \quad (6)$$

If $p > q$ the function does not vary with time and the motion is said to be over damped. If $p < q$, s_1 and s_2 become complex numbers and may be written

$$s_1 = -p + i\sqrt{q^2 - p^2} = -p + i\eta$$

$$s_2 = -p - i\sqrt{q^2 - p^2} = -p - i\eta$$

where

$$\eta = \sqrt{q^2 - p^2}$$

and the general solution of equation (3) becomes

$$x = C_1 e^{(-p+i\eta)t} + C_2 e^{-i\eta t}$$

$$= e^{-pt}(C_1 e^{i\eta t} + C_2 e^{-i\eta t})$$

$$= e^{-pt}(A \cos \eta t + B \sin \eta t) \quad (7)$$

$$A = C_1 + C_2 \quad \text{and} \quad B = i(C_1 - C_2)$$

The relationship between x and t is shown in Fig. (2) when $x = x_0$

and $v = 0$ at $t = 0$

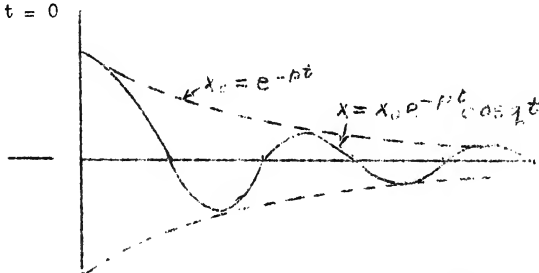


FIG. 2 - FREE VIBRATION WITH DAMPING

As shown in Fig. 2, Equation (7) is a function representing the free vibration portion of equation (1).

A particular solution of equation (1) may be obtained by assuming²⁴

$$x = A \cos \omega t + B \sin \omega t \quad (8)$$

where A and B are constants to be determined then

$$\frac{dx}{dt} = B \omega \cos \omega t - A \omega \sin \omega t$$

$$\frac{d^2x}{dt^2} = -B \omega^2 \sin \omega t - A \omega^2 \cos \omega t$$

and substituting in equation (1) the above functions gives

$$\begin{aligned} & (A q^2 + 2 B \omega p - A \omega^2) \cos \omega t + (B q^2 - 2 A \omega p - B \omega^2) \sin \omega t \\ & = \frac{F}{m} \sin \omega t \end{aligned}$$

By equating coefficients

$$\left. \begin{aligned} B q^2 - 2 A \omega p - B \omega^2 &= \frac{F}{m} \\ A q^2 + 2 B \omega p - A \omega^2 &= 0 \end{aligned} \right\}$$

and solving for A and B

$$\begin{aligned} A &= -\frac{2 \omega p F/m}{(q^2 - \omega^2)^2 + (2 \omega p)^2} \\ B &= -\frac{(q^2 - \omega^2) F/m}{(q^2 - \omega^2)^2 + (2 \omega p)^2} \end{aligned}$$

Placing $A = C \sin \phi$

$B = C \cos \phi$

equation (8) may then be written $x = C \sin(\omega t + \phi) \quad (9)$

where

$$C = \sqrt{A^2 + B^2} = \frac{F/n_1}{\sqrt{(q^2 - \omega^2)^2 + (2\omega p)^2}} \quad (10)$$

$$\tan \phi = \frac{A}{B} = \frac{2\omega p}{q^2 - \omega^2}$$

Equation (9) is a particular solution of equation (1) having the same frequency as the external force. The general solution of equation (1) becomes

$$X = e^{-pt} (A \cos \omega t + B \sin \omega t) + C \sin(\omega t + \phi) \quad (11)$$

which is a combination of a damped free vibration as shown in Fig. (2) plus a forced vibration of the same frequency as the force, $F \sin \omega t$, but lagging the external force by a phase angle ϕ .

It is to be noted in equation (10) that when $p = \frac{c}{m}$ is large, viscous damping will reduce the free vibration to a small value, if the exciting force maintains a constant frequency.

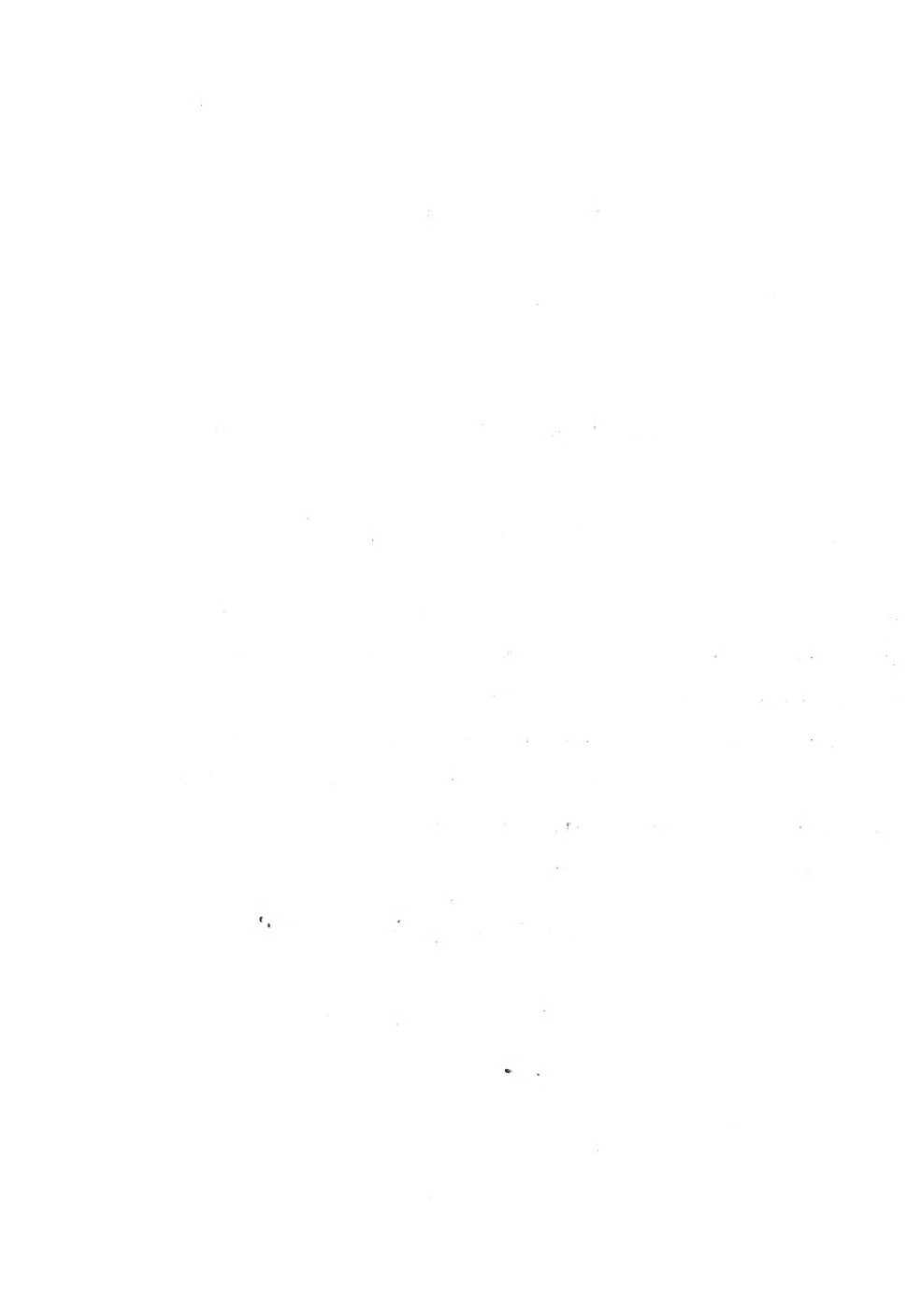
Equation (10) can be written

$$C = \frac{F/n_1}{\sqrt{(q^2 - \omega^2)^2 + (2\omega p)^2}} = \frac{F/p}{\sqrt{(1 - \eta^2)^2 + 4\eta^2}} \quad (12)$$

$$\eta = \frac{\omega}{q} \quad \text{if } p = \frac{c}{q} = \frac{c\sqrt{m}}{2m\sqrt{k}} = \frac{C}{2\sqrt{mk}}$$

Defining the amplification factor as

$$Q = \frac{1}{\sqrt{(1 - \eta^2)^2 + 4\eta^2}}$$



as the denominator becomes a minimum the amplification will be a maximum. This occurs when

$$\eta = 0 \quad \text{or} \quad \eta = \sqrt{1 - 2p^2}$$

or

$$Q_{max} = \frac{1}{2p\sqrt{1-p^2}}$$

2. Beats. If the free vibration is not eliminated by viscous damping a beat phenomena will be produced, in a system of forced vibrations, when the frequency of the free vibration differs from that of the forced vibration.

If two harmonic motions possessing different frequencies are considered²⁴

$$f_1(t) = A \cos \omega_1 t$$

$$f_2(t) = B \cos \omega_2 t$$

where $\omega_1 > \omega_2$,

they will produce a motion

$$X = C \cos(\omega t + \phi) = A \cos \omega_1 t + B \cos(\omega_1 - (\omega_1 - \omega_2)t) \quad (14)$$

and

$$C \cos \phi = A + B \cos(\omega_1 - \omega_2)t$$

$$C \sin \phi = -B \sin(\omega_1 - \omega_2)t$$

then

$$C = \sqrt{A^2 + B^2 + 2AB \cos(\omega_1 - \omega_2)t} \quad (15)$$

$$\tan \phi = - \frac{B \sin(\omega_1 - \omega_2)t}{A + B \cos(\omega_1 - \omega_2)t} \quad (16)$$

Equation (15) indicates that the amplitude is a variable oscillating between $(A-B)$ and $(A+B)$ having a beat frequency

$$n = \frac{\omega_1 - \omega_2}{2\pi} = n_1 - n_2$$

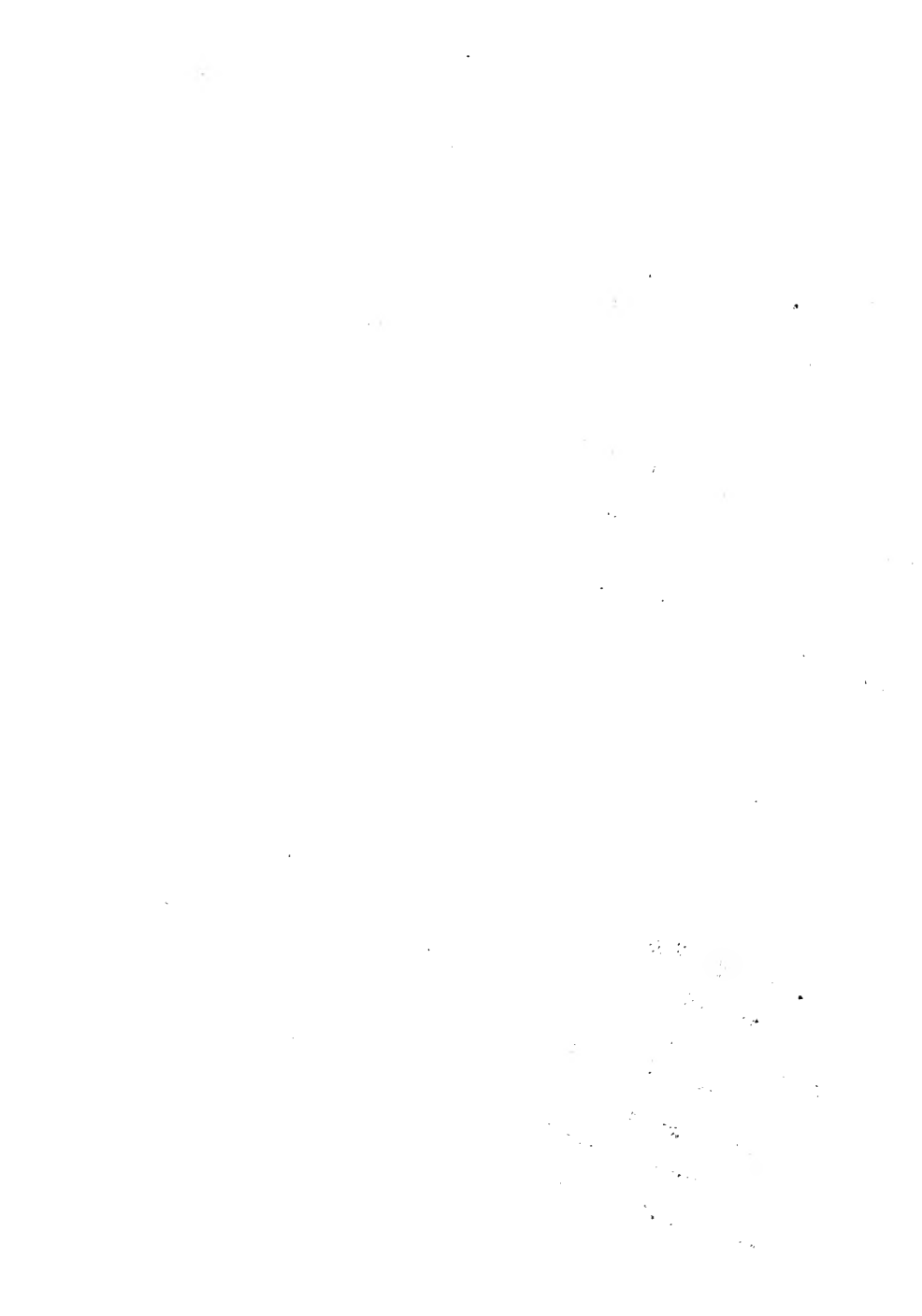
This is shown graphically in Fig. (3)



FIG. 3 - BEAT FREQUENCY

3. Reflected Waves. When a wave possessing one degree of freedom is reflected back into a vibrating system the reflected wave will be returned to the system possessing the same frequency as the primary wave but will be reduced in amplitude depending on absorption by the reflecting surface.

Two waves are therefore considered, travelling in opposite directions, with the same frequency, and differing in amplitude.



$$f_1(t) = A \cos(\omega t - \phi) \quad (17)$$

$$f_2(t) = -B \cos(\omega t + \phi) \quad (18)$$

Adding the two waves gives

$$F(t) = f_1(t) + f_2(t) = A \cos(\omega t - \phi) - B \cos(\omega t + \phi) \quad (19)$$

$$F(t) = (A-B) \cos \omega t \cos \phi + (A+B) \sin \omega t \sin \phi \quad (20)$$

If the velocity of the two waves remains constant, the phase angle ϕ will remain constant at a particular point and

$$F(t) = C_1(A-B) \cos \omega t + C_2(A+B) \sin \omega t \quad (21)$$

$$C_1 = \cos \phi \quad \& \quad C_2 = \sin \phi$$

Equation (21) shows that, when $\sin \omega t = 0$, the resultant is equal to a minimum value $(A-B)$ at a point where $C_1 = 1$ and the amplitude will be a maximum $(A+B)$ at a point where $C_2 = 1$. This distance between the two points will be one-fourth a wave length. This relationship is demonstrated in Figs. (4) and (5).

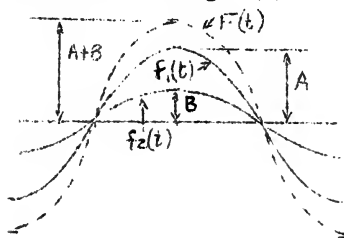


FIG. 4 - WAVE FORM WHEN $C_2 = 1$

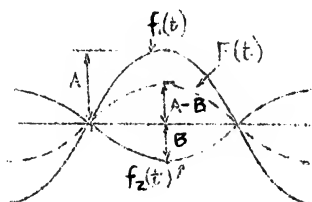


FIG. 5 - WAVE FORM WHEN $C_1 = 1$

If no damping takes place, $A = B$ and the system will contain a standing wave pattern.

4. Wave Length. The wave length of a wave in a vibrating system of harmonic motion is given by

$$\lambda = \frac{v}{n} = \frac{2\pi v}{\omega} \quad (22)$$

where v = velocity of propagation

n = frequency

ω = angular velocity

The forced vibration of a system dynamically loaded is given by

$$\begin{aligned} f(t) &= C \cos(\omega t + \phi) \\ &= C (\cos \omega t \cos \phi - \sin \omega t \sin \phi) \quad (23) \end{aligned}$$

At a given instant of time $t = t_1$, the distance between points where

$\frac{df(t)}{dt} = 0$ will be equal to $\frac{\lambda}{2}$ or

$$\frac{df(t)}{dt} = -\cos \phi \sin \omega t + \sin \phi \cos \omega t = 0 \quad (24)$$

Equation (24) shows that this occurs at points where

$$\left. \begin{aligned} \cos \phi &= 0 \\ \cos \omega t &= 0 \end{aligned} \right\} \text{ at a point at the same instant of time } t = t_1$$

when

$$\left. \begin{aligned} \sin \phi &= 0 \\ \sin \omega t &= 0 \end{aligned} \right\} \text{ at a second point at } t = t_1$$

This relationship is shown graphically in Fig. (6)

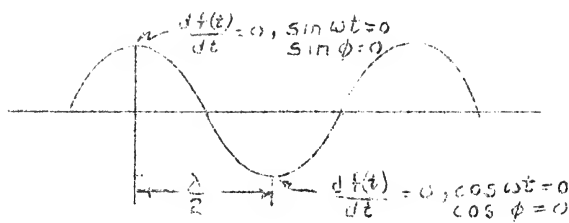


FIG. 6 - WAVE FORM ALONG AXIS OF PROPAGATION AT $t = t_1$

B. Characteristics of Measuring Equipment

1. Nature of waves to be measured. Forces produced by dynamic loading in a visco-elastic medium may be considered to result in two types of phenomena which account for the elastic and plastic properties.

Consolidation occurs with dynamic loading of sandy soils and can be expected to proceed until a compaction is obtained which is a function of the loading force and is independent of the number of loadings which occur after an equilibrium condition is obtained. A fine grained cohesive soil might also be compacted, provided a sufficient number of loadings are applied to obtain the necessary drainage conditions for consolidation. It appears that such consolidation would be a function of the average load applied and time.

Dynamic loading will produce three types of waves in the soil. In order of decreasing velocity they are; compression waves, distortion or shear waves, and surface waves.² These waves are associated with the elastic properties of the soil.

One of the objectives of this study was to reduce or eliminate all but the compression wave. If a tube of soil as shown in Fig. 7 is loaded dynamically with a sinusoidal force, the shear waves developed

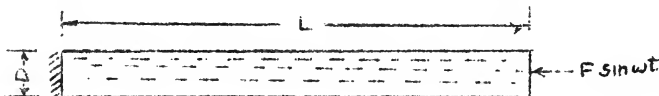


FIG. 7 - SCHEMATIC LOADING OF A SOIL TUBE

in the soil will be generated in a transverse direction to the length of the tube as a function of the angle of internal friction of the soil. If the wave front of the compression wave is a plane oriented transverse to the tube axis, at any point in the tube particles of soil will be moving parallel to each other with the same velocity and acceleration at any instant of time, except at the inside surface of the tube. Shear waves can develop only when a wave is propagated into a zone where the shear strength of the soil can be mobilized. The system of Fig. 7, therefore, can develop shear waves only at the inside surface of the tube provided that the soil grains are in the form of columns of individual particles possessing elastic properties. Such is not the case in a real soil. The compression wave in propagating itself must develop shear in the soil in order for the force to be transmitted from grain to grain and a shear force or wave is thus developed in the soil. The surface waves are eliminated by enclosing the soil in a tube thus preventing unrestrained surface movement of the particles. Assuming that the energy transmitted by a compression wave is large compared to the energy of the shear wave, the compression

wave must receive prime consideration in developing a measuring system.

If a flat plate is placed parallel to plane a-b in Fig. 7 the plate will deflect as it is loaded as shown in Fig. 8.

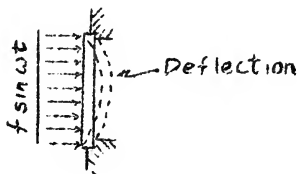


FIG. 8 - DEFLECTION OF A VERTICAL PLATE

If a plate is placed perpendicular to the axis of the tube, it will be loaded as a function of the angle of internal friction for the soil and friction between the soil and the surface of the plate as shown in Fig. 9.

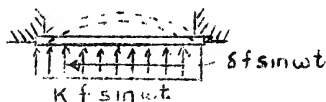


FIG. 9 - DEFLECTION OF A HORIZONTAL PLATE

where $K = \tan^2 (45 - \phi/2)$

ϕ = angle of internal friction

δ = coefficient of friction between plate and soil

The unit loading on the plate in Fig. 10 producing deflection is

$$K \frac{F}{A} \sin \omega t = f \sin \omega t \tan^2 (45 - \frac{\phi}{2})$$

2. Characteristics of Flat-Diaphragm Pressure Receivers.²⁷ Pressure

receivers of the flat diaphragm type utilize the principle that deflection of a circular plate is proportional to the pressure applied to the diaphragm or plate.

The static characteristics of a flat diaphragm can be expressed in terms of sensitivity defined as the ratio of a small output change to small input change when all other variables are held constant. For a flat diaphragm, sensitivity of the pressure receiver is equal to the ratio of displacement of the center of the diaphragm to the applied pressure.

Metallic diaphragms are affected by four input quantities:

- (1) Pressure applied to the outer surface of the diaphragm.
- (2) Linear acceleration of the diaphragm support.
- (3) Temperature gradient across the two surfaces of the diaphragm.
- (4) Average temperature of the diaphragm material.

Pressure is the only desired input of the above factors. The other quantities result in deflections which cannot be separated from pressure input.

In order to utilize a flat diaphragm in a vibrating system the dynamic characteristics must be considered. Dynamic characteristics will depend primarily upon the undamped natural frequency. A plot of this characteristic as a function of thickness, h , for a diaphragm having a radius of $1/2$ inch is given in Fig. 10. Deflection sensitivity is plotted as a function of thickness in Fig. 11. This figure shows the tremendous loss in sensitivity that occurs when attempts are made to increase response by increasing the thickness.

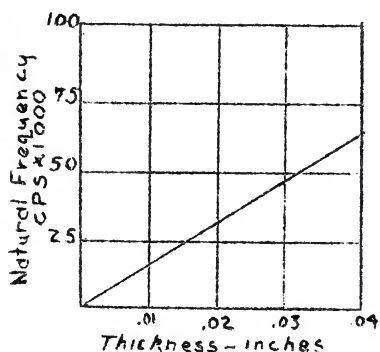


FIG. 10 - NATURAL FREQUENCY OF
1/2" DIAMETER DIAPHRAGMS

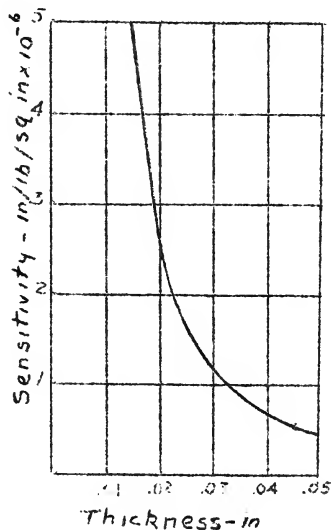


FIG. 11 - SENSITIVITY OF 1/2"
DIAMETER DIAPHRAGM

Acceleration pressure sensitivity is the ratio of acceleration - deflection sensitivity to pressure-deflection sensitivity. It has dimensions of pressure divided by acceleration and represents the pressure required to produce the same diaphragm output deflection as a unit acceleration of the support.

The curves in Fig. 12 show that acceleration pressure sensitivity varies as a function of the forcing frequency of the support and the natural frequency of the diaphragm. It increases with increased thickness and frequency ratio.

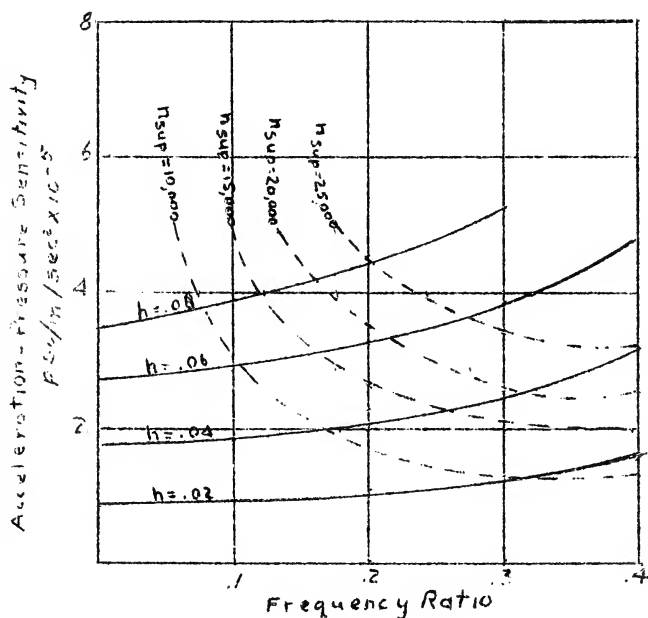


FIG. 12 - ACCELERATION SENSITIVITY

It may be concluded that the undesirable effects of support acceleration are considerably reduced by maintaining a low ratio of support forcing frequency to the natural frequency of the diaphragm.

Temperature gradients across the two surfaces of the diaphragm are very small in this work and will not be considered. The average temperature of the diaphragm will also be maintained at a low value and loss of sensitivity due to this factor will be insignificant.

3. Diaphragm Displacement Measurement. Many methods may be utilized in transposing deflection of the diaphragm into an electrical quantity which can be measured. Of these methods, three general types of systems were considered to be practical. The deformation of crystals in a manner similar to a phonograph pickup may be used. A second method utilizes changes in an electromagnetic field to produce a signal. The third method utilizes S-R-4 Strain Gages which are based on a resistance change principal.^{3,9} All these devices require amplification of a signal.

The S-R-4 Strain Gages were selected as a simple device to be utilized in the studies. They are easily mounted and provide linear response without adding appreciable weight to the diaphragm.

Deflection of the gage produces a change in resistance across the gage. The simplest circuit in which such a gage can be used to record deflection from dynamic loading is shown in Fig. 13.¹

This circuit is coupled with an amplifier and the output is observed with an oscilloscope.

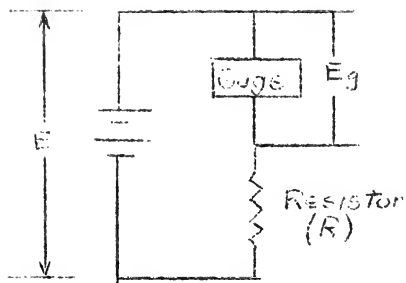


FIG. 13 - STRAIN GAGE CIRCUIT FOR ONE GAGE

A circuit which has many advantages over the above circuit utilizes a Wheatstone bridge. If two gages are used in the bridge as shown in Fig. 14 an unbalance in the bridge will result from deflection of one or both of the gages.

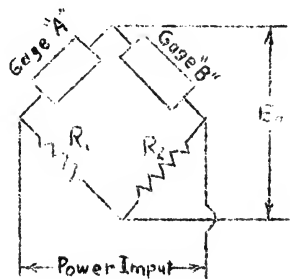


FIG. 14 - WHEATSTONE BRIDGE CIRCUIT FOR TWO GAGES

This circuit has two main advantages. First, if both gages are mounted so that they receive the same temperature, change in resistance in one gage due to temperature will be compensated for by changes in the second gage. A second advantage of the above circuit may be realized by mounting the two gages on either side of the diaphragm. Thus a deflection causing a tension strain in one gage will produce a compression strain on the other gage. This will result in an amplification of the signal by a factor of two.

In this circuit the unbalance of the bridge is given by

$$dE_o = \left[\frac{R_A R_1}{R_A + R_1} + \frac{R_B R_2}{R_B + R_2} \right] I K dS$$

where E_o = Bridge output voltage

R_A and R_B = Resistance of the Strain Gages

R_1 and R_2 = Resistance of the Bridge Legs

I = Current through each gage

dS = change in strain applied to the gage (inches per inch)

K = Gage Factor $\left(\frac{d R_G}{R_G dS} \right)$

A schematic diagram of the complete circuit is shown in Fig. 15.

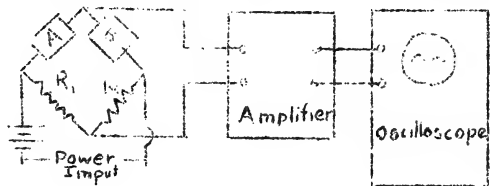


FIG. 15 - SCHEMATIC OF PRESSURE TRANSDUCER CIRCUIT

PART III

MATERIALS AND APPARATUS

A. The Loading System

1. Sinusoidal Pulse. A system designed to produce dynamic wave forces in soil, and to provide a method for analysis of the waves thus produced, must be capable of loading a mass with a force which varies sinusoidally with time. This statement implies that the pulses developed must not behave as solitary waves or have a force time relationship similar to a shock pulse or series of shock pulses.

Satisfaction of the above condition requires that a loading device maintain contact with a mass being loaded throughout the cycle. Three methods were considered for meeting this requirement taking into account that horizontal loading is a requirement of the system.

The first of these systems was an oscillator of the "Degebo" type.⁷ This oscillator utilizes the eccentricity of two masses rotating in opposite directions as shown in Fig. 16.

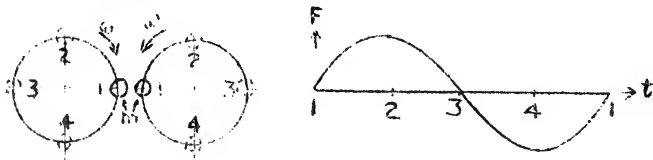


FIG. 16 - MOTION OF THE DEGEBO OSCILLATOR

When the two masses m , are in position 1, no vertical force is being developed and the horizontal forces cancel each other. As position 2 is reached both masses act together producing the vertical force shown at point two on the curve. At position 3 the horizontal forces again cancel and at position 4 they reinforce each other.

In order for an oscillator of this type to transmit the force developed to a soil mass, the weight of the oscillator must be greater than the maximum force produced. At low speeds this is no problem, but, as the circular frequency of the oscillator increases, the developed maximum force increases. When utilizing the oscillator for vertical loading, use may be made of the weight of the oscillator. When a force is desired in a horizontal direction the weight of the oscillator cannot contribute a force so as to maintain surface contact with the soil mass.

The required sinusoidal force can be developed in a horizontal direction by a system such as shown in Fig. 17.

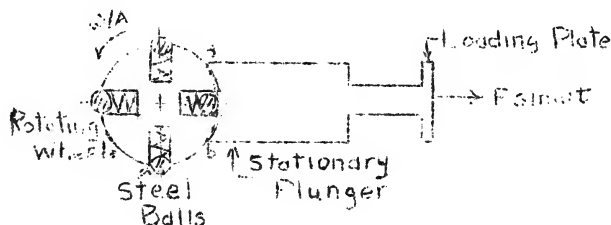


FIG. 17 - DYNAMIC LOADING WHEEL

The system consists of a rotating wheel with symmetrically spaced holes drilled into the rim. Each of the holes has a steel ball loaded by a spring. As the ball engages the stationary plunger, it will load the plunger by centrifugal force in addition to the force developed in compressing the spring. The variation of the loading force as a function of time will depend on the shape of the edge, a-b, of the plunger.

The main disadvantage of this system is that the weight of the ball must be relatively small and the force thus produced will be small at low wheel rotation speeds. It has an advantage in that the mass is not a factor in maintaining contact as long as a proper spring constant is chosen. The rotational velocity of the wheel need be only a fraction of the desired pulse frequency depending on the number of steel balls used.

A third system was adopted as a means to develop the sinusoidal force, which avoids some of the limitations of the other two methods and is shown in Fig. 18.

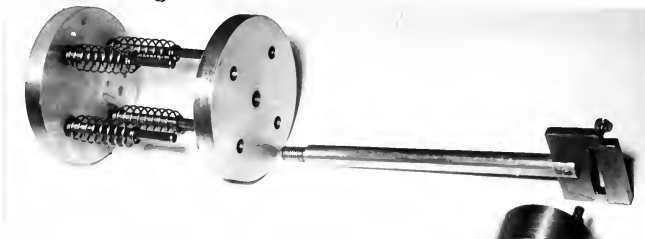


FIG. 18 - PLUNGER TYPE DYNAMIC LOADER

In this system an eccentric pin acting on a "Scotch Yoke" develops a horizontal motion which varies sinusoidally with time. The yoke may be considered to act as a lever arm of infinite length. The plunger loads the springs which bear on the loading plate producing a sinusoidal force on the loading plate. The maximum amplitude of the force will depend on the spring constant, the circular frequency of the eccentric pin, and the amount of deflection produced in the spring. The pulse will not be distorted unless the system operates at or near the natural frequency of the spring. A beat frequency will be developed as the forcing frequency approaches the natural frequency of the spring as discussed in Part II - A.

2. Design of the Loading System. The mechanism described above and shown in Fig. 18 was the loading system chosen for the studies. The loading plate and plunger were made of aluminum. The "Scotch Yoke" was originally made of $1/4$ aluminum plate, however, excessive wear resulted and a steel yoke had to be substituted.

Aluminum was used in an attempt to reduce the weight of the plunger as much as possible. The loading plate and plunger were 4" in diameter with just enough clearance for the inside diameter of the tube. The guide pins for the springs were steel, $1/4$ " in diameter. The eccentric pin had a $1/2$ " radius which produced a 1" plunger throw. The slide bushings were elliptical return ball bearing bushings and were used to guide the plunger shaft of $1/2$ " diameter. This equipment is shown in Fig. 19.

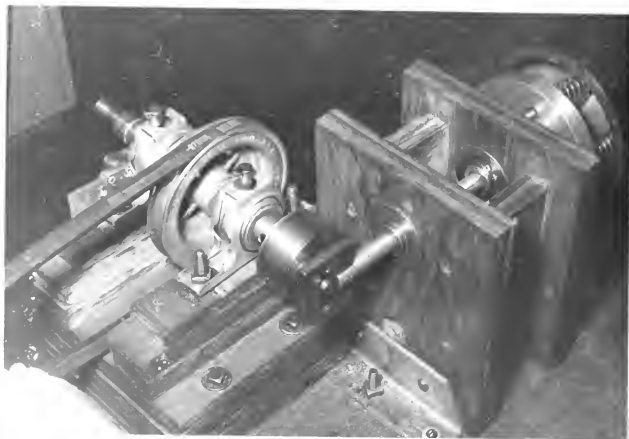


FIG. 19 - THE LOADING SYSTEM

As shown in the figure, power for this unit is provided by a $1/4$ H.P. capacitance motor operating on 110 V-A.C. at 1725 revolutions per minute. A capacitance motor was used in an effort to provide constant speed under a varying load.

The motor was connected by a belt to a three position pulley which provided three eccentric speeds of 1740, 1305, and 870. The pulley shaft was mounted on self aligning pillow blocks which contained bronze sleeve bushings.

B. The Soil Tube

1. Function of the Tube. Soil may be loaded dynamically in a number of ways. An evaluation of the forces produced will depend upon the physical geometry utilized. As discussed in Part II - B, the tube

was selected as a method for reducing the number of forces being studied to what is assumed to be a compressive wave. Consideration was given to suspending the tube so as to reduce the possibility of vertical forces entering into the system. A limitation of this idea was the necessity for providing a mounting for the loading system. It is necessary to rigidly mount the loading system and to provide a connection to the tube. This, then, dictated the need for rigidly mounting the tube itself. It must be recognized that vertical forces developed in such a system cannot be accounted for.

The horizontal orientation of the tube was selected in order to reduce the effect of a variation of intergranular pressure throughout the length of the tube. Propagation velocity can be expected to change with intergranular pressure. This is indicated by the velocities reported by Bernhard and Finelli.²

2. Tube Design. The soil tube is constructed of "Orangeburg" pipe, 4" in diameter. Two 5' sections are used with a joint in the center to facilitate loading with soil. One inch holes are provided at twelve inch intervals along the length of the tube to provide access for the pressure transducer.

An end block is provided to resist horizontal movement of the tube and soil. The tube is mounted on wood cradles, as shown in Fig. 20, which are supported on a wood platform.

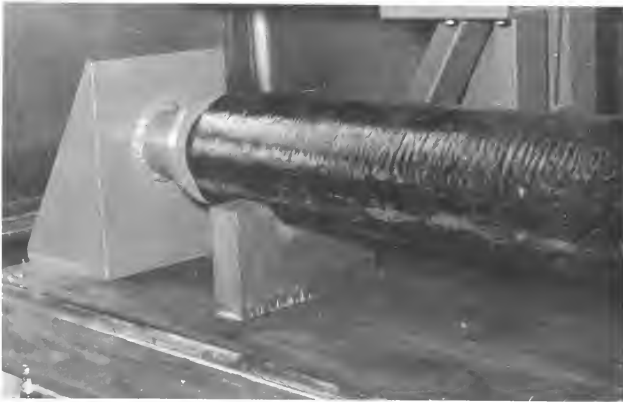


FIG. 20 - THE SOIL TUBE

C. The Pressure Transducer

1. Obtaining Sensitivity. Of great importance in a study of this type is the design of an instrument which will provide faithful response to the forces being investigated. A pressure pickup or transducer possessing proper characteristics can fulfill this function based on the principal that the soil loads the transducer in direct proportion to the area exposed.

If this requirement is met, several methods for translating such pressure into electrical impulse can be used, depending on the degree of sensitivity desired. The ease and economy in using SR-4. Strain Gages makes this task much less complicated.

Various electrical circuits which may be employed using SR-4 Strain Gages have been discussed in Part II - B. However, they may be installed in several ways in order to obtain increased sensitivity. A compromise must be made in the selection of the method.

If it is possible to accept a lower level of sensitivity, the method of mounting shown in Fig. 21 can be used. This method has the decided advantage of protecting the delicate strain gage from mechanical damage by mounting the gage on the side of the diaphragm which is not in contact with the soil. Fig. 21 shows an "active gage" which



FIG. 21 - MOUNTING FOR ONE ACTIVE GAGE

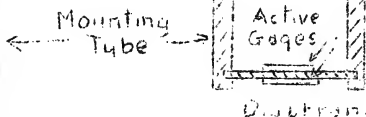


FIG. 22 - MOUNTING FOR TWO ACTIVE GAGES

is stressed by the load and a "dummy gage" used for temperature compensation. The "dummy gage" is mounted on the wall of the mounting tube.

Sensitivity of the transducer can be doubled by mounting a gage on each side of the diaphragm as shown in Fig. 22. The gage mounted on the outside is subject to mechanical damage from the soil and must, therefore, be protected. Quartz grains in sand will short out connection wires and these wires must also be protected.

If greater increases in sensitivity are required the strain gages can be mounted on a compression ring as shown in Fig. 23. Distortion

of the ring by deflection of the plate will provide greater sensitivity and an added factor of safety from mechanical damage. Such a method is obviously much more complicated than mounting directly on the diaphragm. The sensitivity of the method shown in Fig. 23 can be

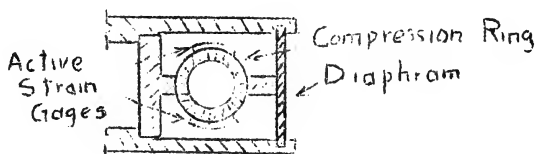


FIG. 23 - STRAIN GAGES MOUNTED ON COMPRESSION RING

further increased by utilizing an elliptical shape for the compression ring and by mounting two additional strain gages on the inside surface of the ring.

2. Design of the Pressure Transducer. The first pressure transducer built was of the type shown in Fig. 22, however, instead of mounting the "dummy gage" as shown, it was mounted on a plate suspended inside the mounting tube. This was found to be very inadequate due to accelerations developed in the dummy plate.

A second, more successful, design was that of the type shown in Fig. 22. The gage mounted on the exterior surface of the diaphragm was subjected to considerable mechanical damage.

The diaphragm itself was .01 inches thick and had a $1/2$ inch diameter. This diaphragm constructed of steel has a natural frequency of 20,000 cycles per second and a sensitivity of about .0225 inches per pound per square inch.

The mounting tube was adapted from an amphenol connector. The connecting ring was used to retain the diaphragm. Wires leading from the outside strain gage passed through a hole in the side of the mounting tube. The transducer is shown in Fig. 24.

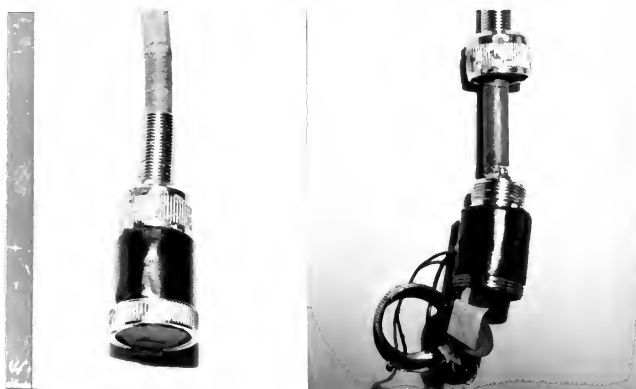


FIG. 24 - THE PRESSURE TRANSDUCER

Plastic tape was used to insulate connecting wire from the outside surface of the diaphragm to prevent mechanical damage to the gage. A thin sheet of plastic was placed over the end of the transducer to prevent sand grains from entering the mounting tube and shorting connections. The plastic tape over the diaphragm did not reduce sensitivity because the load was transferred directly to the diaphragm. The response of the diaphragm was not affected in the range of frequencies encountered in these studies. It would be difficult to assign a value to this factor.

All lead wires from the transducer to amplifier and to the oscilloscope were shielded. A great deal of stray current is always present and satisfactory performance demands effective shielding.

3. The Transducer Support. One of the most important factors in measuring wave forces is loss of sensitivity and deflection of the diaphragm resulting from vibrational motion of the support. Part II - B indicated sensitivity losses from support acceleration. Fig. 12 is based on loss of sensitivity with the assumption that the force, $F \sin \omega t$, is unaffected by support accelerations. This assumption holds when fluid pressure is the quantity being measured, however, coupled with losses of the type shown in Fig. 12, a diaphragm in contact with soil will be subjected to additional pressure force produced by the vibrating support. This pressure would be developed by a vibrating support if there were no additional forces in the soil. If the support is allowed to vibrate, a beat frequency will be produced in the diaphragm.

In order to avoid such accelerations it is necessary to provide a support for the transducer which will hold support accelerations to a minimum. An attempt to satisfy this requirement was made by suspending the transducer from a steel bar supported at a distance from the base of the tube support. This system is shown in Fig. 25.



FIG. 25 - TRANSDUCER SUPPORT

4. Amplification and Recording. Signals which are generated by unbalancing the Wheatstone Bridge of the transducer circuit must be amplified before they can be recorded by an oscilloscope.

This amplification was accomplished by a Norwood Control Engineering Co., Model 5AC, Pressure Monitor. The Monitor has a self contained power supply consisting of a dry batteries, an amplifier with a voltage gain of 100, two legs of the bridge circuit for the transducer, and a calibration circuit. The bridge has a variable resistor to permit balancing.

The amplifier has a frequency response from $1/2$ CPS to 25,000 CPS with + or - 2%. The circuit diagram for this unit is shown in Fig. 26.

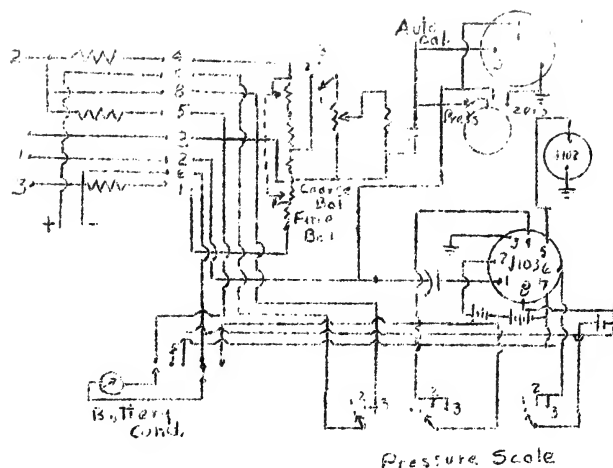


FIG. 26 - CIRCUIT DIAGRAM FOR 5 AC PRESSURE MONITOR

The oscilloscope was a General Electric Model 4STZB1. The unit has a medium persistence screen, 5UP1, a voltage range of from 5 millivolts to 500 volts, and a 10 to 1 gain may be obtained from the vernier gain control. A sweep range of from 2 cycles to 30 kilocycles is available with the sweep either recurrent or triggered. This equipment has a blank input post for external blanking or Z-axis modulation. It has a sensitivity of 10 mv/inch for the vertical sweep.

The oscilloscope screen was photographed with a Dument Poloroid Oscilloscope Camera with an f 2.8 lens.

The amplifying and recording equipment are shown in Fig. 27.

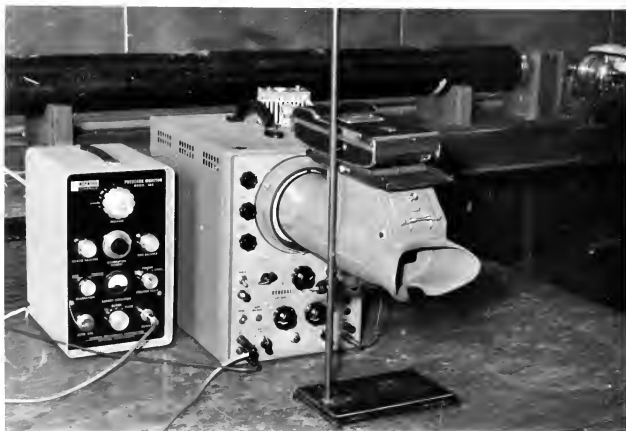


FIG. 27 - AMPLIFYING AND RECORDING EQUIPMENT

5. The Marking Pulse. The determination of wave length and velocity of vibrating forces in soils can be accomplished by measuring the phase lag of the forced vibration at various points in the soil tube. Fig. 6 shows the wave form, at a particular instant of time, along the axis of the tube. This wave form will be repeated once each revolution of the eccentric pen or once during each cycle of the forcing frequency.

The wave form of Fig. 6 can be traced along the axis of the tube by superimposing a marking pulse on the wave form picked up by the pressure transducer at points spaced along the tube. The marking pulse must occur at the same instant during each cycle of the forcing load.

Such a pulse was produced by utilizing a set of automobile ignition points being made and broken by a can attached to the shaft on

which the eccentric is mounted. This is shown in Fig. 28. The circuit

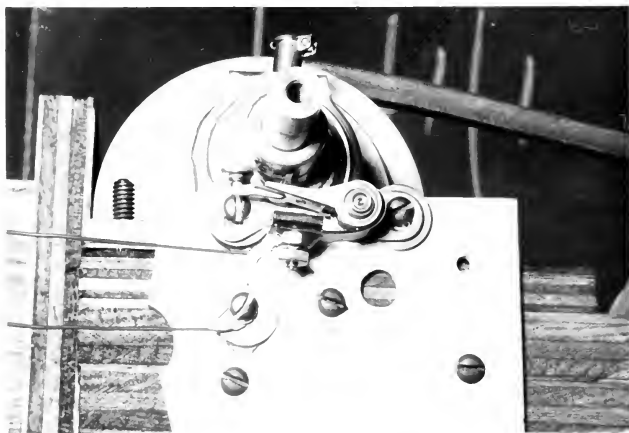


FIG. 28 - THE MARKING PULSE CONTACTS

diagram for the marking pulse is shown in Fig. 29. The value of the

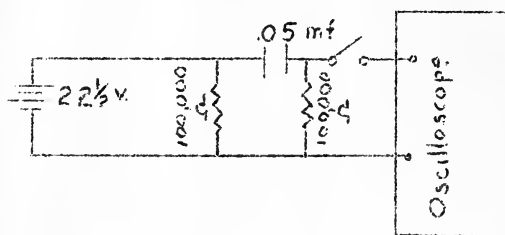


FIG. 29 - CIRCUIT DIAGRAM FOR MARKING PULSE

capacitance was obtained experimentally by selecting a size that would produce a short pip in the circular frequency range used.

By observing the position of the pulse on the wave form, it is possible to determine the phase lag, ϕ , of the wave form in the soil.

PART IV

EXPERIMENTAL PROCEDURE

A. Balancing the Bridge

Fig. 14 shows the Wheatstone Bridge, two legs of which are SR-4 Strain Gages mounted on the diaphragm and two resistors R_1 and R_2 which are contained on the pressure monitor.

One of the resistors in the monitor is a variable resistor or rather a series of resistors which provide a fine and coarse balance for the bridge.

The transducer is connected to the monitor which is, in turn, connected to the vertical input of the oscilloscope. With atmospheric pressure on the diaphragm the bridge is ready to be balanced.

The calibration circuit on the monitor provides a short pulse or pip which is used for balancing. The pulse replaces the signal being observed on the oscillograph screen. The coarse and fine balance controls change the resistance of one leg of the bridge and these controls are used to bring the vertical displacement of the pulse into line with the signal from the transducer in a static condition. At this point the bridge circuit is balanced. Any vertical displacement of the signal, after this balancing, will be the result of pressure on the diaphragm.

B. Filling the Soil Tube

A ten foot length was chosen for the soil tube in order to obtain sufficient tube length to observe changes in the length of the

compression wave as it progresses down the tube. It is also necessary to measure wave length after the free vibration has been damped. All of these considerations dictated the need for a soil tube as long as is practical.

The two five foot sections of the tube are joined at the center by a sleeve joint. Sand was placed in each section of the tube, with the access holes for the pressure transducer stopped off, by funneling the soil into the tube. Circular wood blocks were placed in the end of the tube before filling to retain the sand. The section of the tube in which the loading plate is located was fitted with a 4 inch rubber diaphragm. The diaphragm is necessary to prevent fine particles soil from wedging around the loading plate. If this wedging occurs, the loading impulse will be absorbed by friction.

After the two sections of the tube were placed in position, additional sand was added through the access holes adjacent to the joint in order to fill a void space at the joint.

The loading system was operated for an initial period to compact the sand under the loading force.

C. Measuring Wave Length

The soil tube was provided with six access holes along the top surface of the tube to place the pressure transducer directly in contact with the soil. These access holes were filled with a small amount of fine sand having a uniform particle size distribution. The function of the fine sand is to load the diaphragm with an evenly distributed pressure without concentrations as would be produced by large sand grains.

The pressure transducer was placed in an access hole so that it would not have direct contact with the tube. It was clamped in place by the support apparatus shown in Fig. 25. An initial static pressure was applied to the diaphragm by lowering the support clamp. This pressure was maintained at a constant level at each access position by observing the bridge unbalance as described in Part IV - A.

The loading system was turned on and the wave form was observed on the oscilloscope screen. The sweep frequency of the oscilloscope was adjusted so that the wave form was repetative without moving across the screen. This requires the sweep frequency to be adjusted close to the frequency of the wave form in the soil. The position of the marking pulse was observed on the wave form and a picture of the screen was taken.

The forcing frequency, sweep frequency, access position, horizontal gain and vertical gain were recorded for each position. The process was repeated at each of the six positions along the tube at a particular angular velocity of the loading system.

The theoretical wave form was divided into sectors as shown in Fig. 30. The position of the marking pulse was recorded as occurring at or between these divisions.

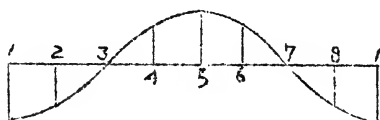


FIG. 30 - SECTORS OF THE WAVE FORM

Access position locations were drawn to scale and the theoretical wave height corresponding to the proper marking pulse position was plotted as shown in Fig. 31.

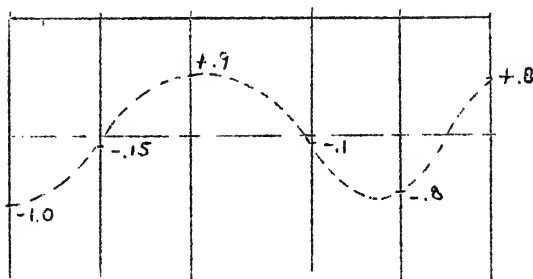


FIG. 31 - WAVE LENGTH PLOT

Access Position 1 indicates that the pulse was located between the fourth and fifth divisions of Fig. 27. The distance between points where the wave height is +1 and -1 will be equal to one-half the wave length.

D. Determination of the Propagation Velocity

With the wave length obtained as demonstrated above for a particular forcing frequency of the loading system, the propagation velocity of the compression wave can be determined from

$$\lambda = \frac{V}{n} = \frac{2\pi V}{\omega}$$

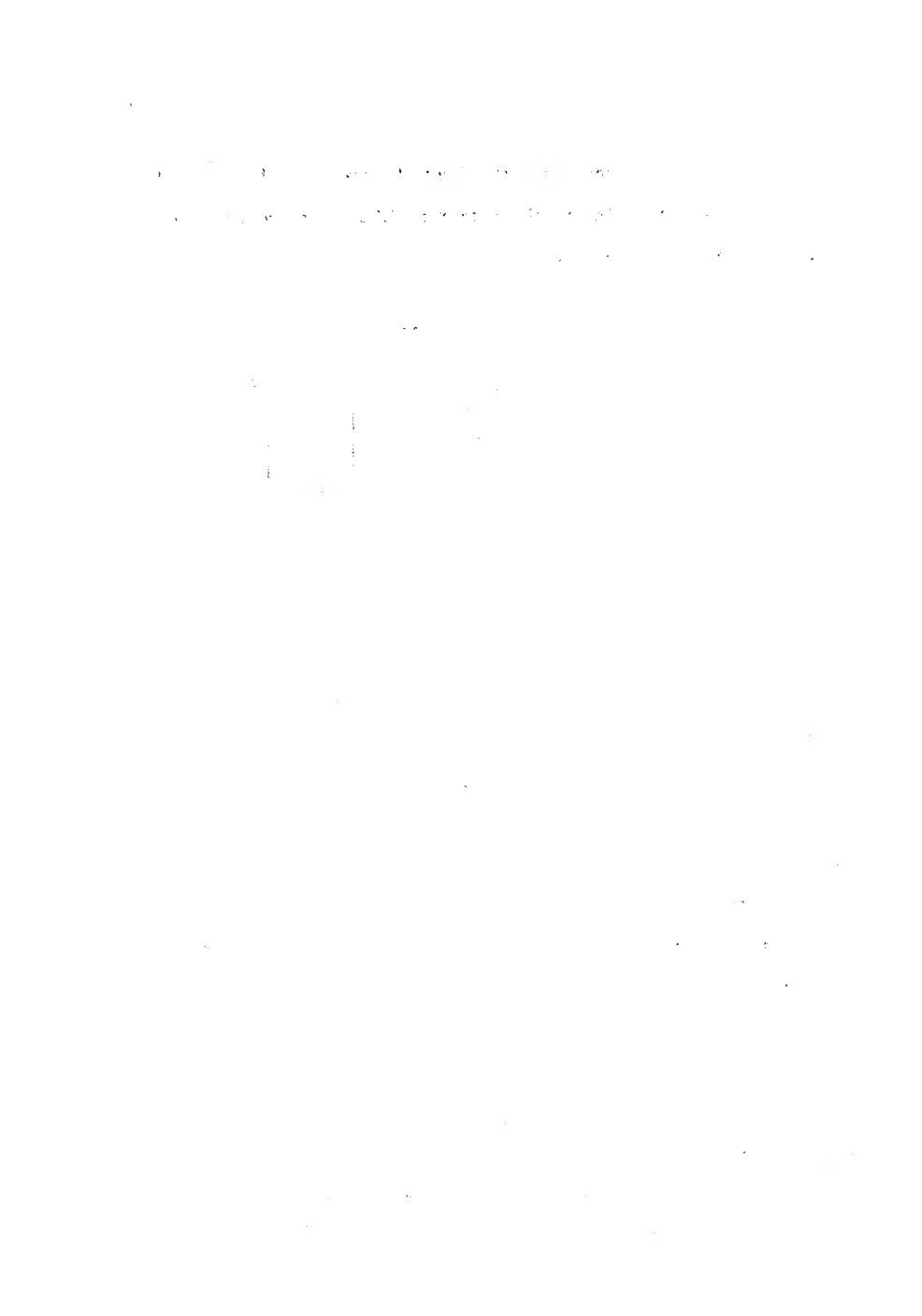
then

$$V = \frac{\lambda \omega}{2\pi} = n\lambda \quad (27)$$

where λ = Wave length

ω = Circular frequency of the loading system

n = Frequency of the system in revolutions per second



PART V

RESULTS

A. Wave Form

The transducer, which was constructed for this study, produced a certain amount of noise when amplification was a maximum. This noise is shown in Fig. 32 taken at a sweep frequency of 150CPS with no load on the transducer.

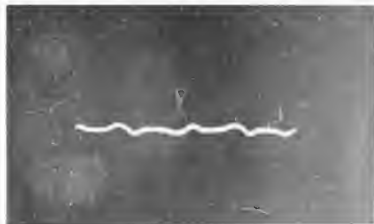


FIG. 32 - PRESSURE TRANSDUCER NOISE

Considerable difficulty was experienced in isolating the transducer so that interference from wave forces in the tube itself would not become a significant factor. An example of a beat frequency which resulted from such interference is shown in Fig. 33. The sweep frequency

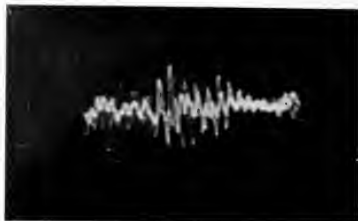


FIG. 33 - BEAT FREQUENCY DUE TO INTERFERENCE

was 6CPS and the forcing frequency was 15CPS. This pattern occurred at position 1 which is located 12 inches from the loading plate.

The nature of this interference is not known. It may have been due to a surface wave at the interface between the soil and the tube or a wave in the tube itself. The fact that the beat is present indicates an interference frequency which differs from the forcing frequency.

All of the observations made at access position 1 showed an extremely short period wave of small amplitude. The frequency of this wave is on the order of 100CPS. This wave may be associated with the natural free vibration of the soil, however, it was not possible to provide a direct correlation.

B. Wave Length

Wave length measurements were made of a Fort Jefferson Sand at forcing frequencies of 14.5, 21.75, and 29.0 cycles per second. A separate plot was made for each run based on visual observations of the screen and the marking pulse position shown on the films. Fig. 34 shows a typical run taken at a forcing frequency of 29 cycles per second.

This figure shows that two wave forms were present on the system. A primary wave is shown at access position 1 with a smaller wave lagging by 90°. The amplitude of the secondary wave is approximately one half the primary wave, but, the frequency is equal to the frequency of the first wave. The velocity of the smaller wave differs from the

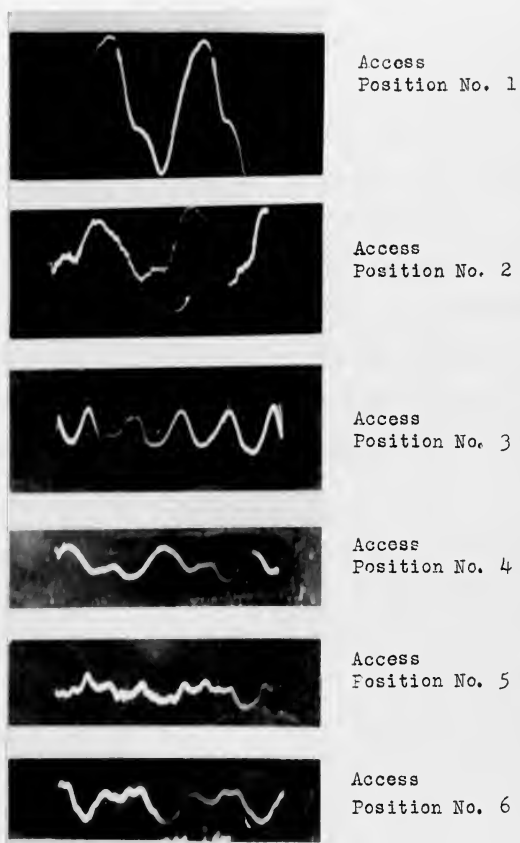


FIG. 34 - TYPICAL WAVE FORM RECORDING AT $n = 29.0$ CPS

main wave as is evidenced by access position 3. At this point, the smaller wave is 180° lagging the primary wave.

Fig. 35 shows a plot of the wave form of the primary wave at a forcing frequency of 14.5CPS, based on both visual and filmed positions of the marking pulse. Fig. 36 shows a similar plot for $n = 21.75\text{CPS}$ and Fig. 37 shows the wave form for $n = 22.00\text{CPS}$. Fig. 38 and Fig. 39 are check runs for $n = 21.75$ and $n = 14.5$ respectively.

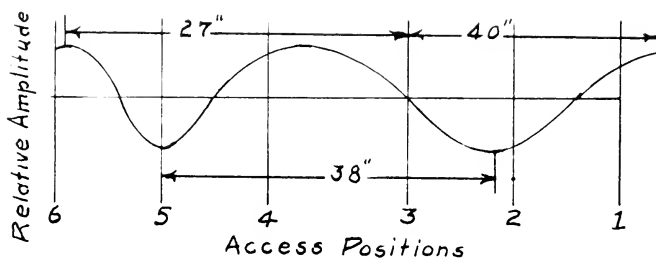
These plots show a shortening of the wave length as the wave progresses down the tube. This shortening process is probably due to increased density close to the loading plate as a result of compaction.

A plot of wave length vs frequency is shown in Fig. 40. This curve shows the greater influence of frequency at lower rotational speeds.

The wave lengths measured in this study are on the order of a tenth of the wave lengths reported by other investigators.^{2,14} This difference can be attributed to a difference in the nature of the waves recorded. Nijboer and Van Der Poel state that vibrations recorded in their study were of the transversal type. The pickup which they used was placed at the surface of the ground and had no external support. Bernhard and Finelli used a pressure transducer which was buried in the soil.

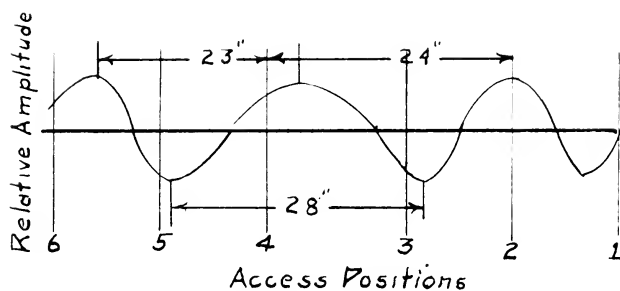
C. Velocity Measurements

The velocity of the waves as computed from equation (27), is shown as a function of frequency in Fig. 41. The velocities thus



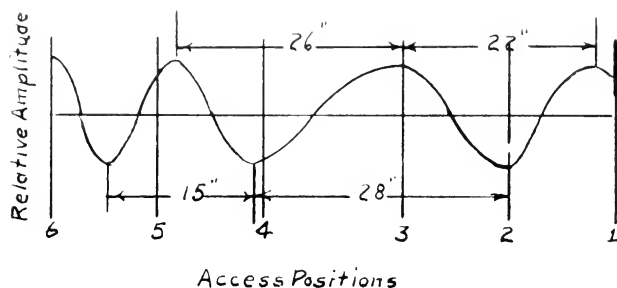
$$V = \frac{35 \times 14.5}{12} = 42.34 \text{ ft./sec.}$$

FIG. 35 - WAVE FORM AT $n = 14.5$ CPS



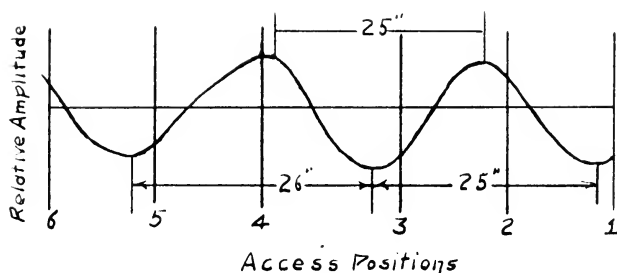
$$V = \frac{35.7 \times 11.75}{12} = 46.4 \text{ ft./sec.}$$

FIG. 36 - WAVE FORM AT $n = 11.75$ CPS



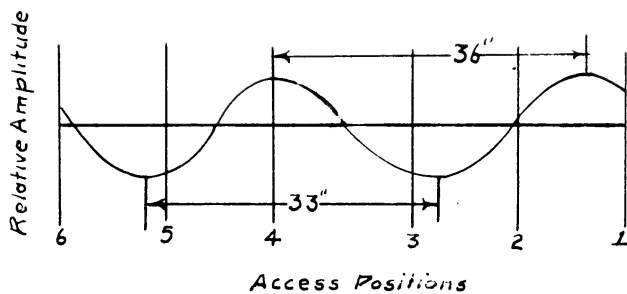
$$V = \frac{24.7 \times 19.2}{1} = 59.6 \text{ ft./sec.}$$

FIG. 37 - WAVE FORM AT $n = 29 \text{ CPS}$



$$V = \frac{5.3 \times 21.75}{12} = 45.6 \text{ ft./sec.}$$

FIG. 38 - WAVE FORM AT $n = 21.75 \text{ CPS}$



$$V = \frac{34.5 \times 14.5}{12} = 41.5 \text{ ft./sec.}$$

FIG. 29 - WAVE FORM AT N = 14.5 CPS

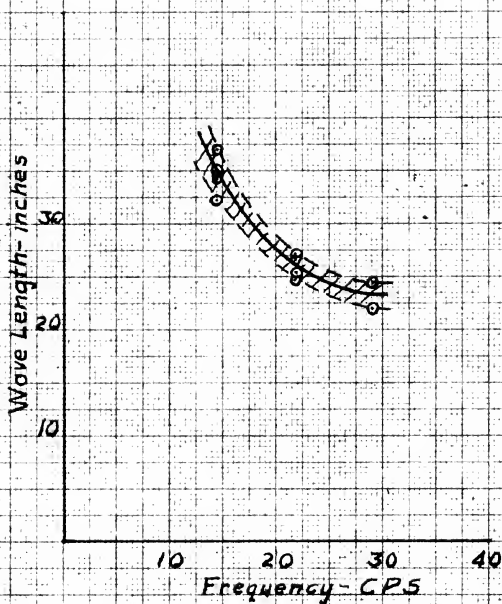


FIG 40 - WAVE LENGTH vs FREQUENCY

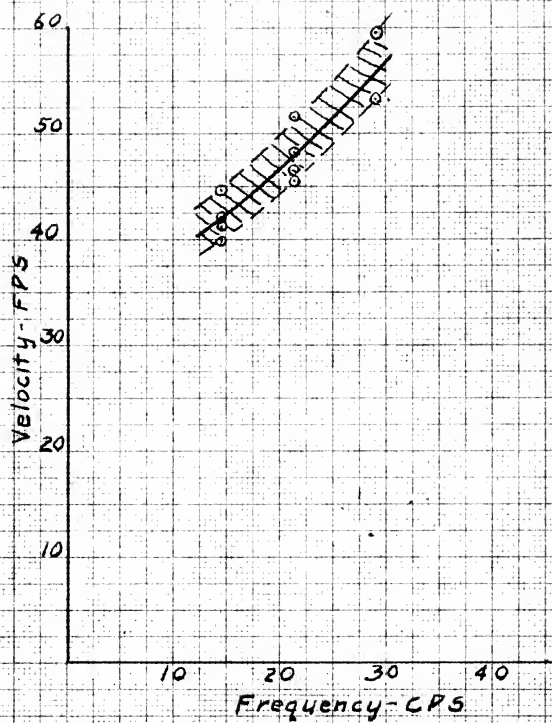


FIG. 41 - PROPOGATION VELOCITY vs FREQUENCY

computed are considerably less than the 500-1000 foot per second range indicated by others.^{2,14,23} The hyperbolic shape of the velocity curve is similar to the curves shown by Nijboer and Van Der Poel.¹⁴

Although the waves measured differ in value from the work cited above it is believed that the relationship between frequency, wave length, and velocity does provide a useful means for evaluation of soil characteristics.

PART VI
DISCUSSION

A. Limitations of the Testing Equipment

The apparatus constructed for this work should be considered as an attempt to develop a method for producing and measuring the forces associated with a compression wave.

The loading device differs considerably from the, now classic, methods which utilize an oscillator of the "Degebo" type. The pulley system does not provide an infinite range of frequencies, a feature which is extremely desirable. It does have an advantage over the oscillator for producing horizontal loading.

The mechanical design of the loading system is not sturdy enough to provide absolutely reliable sinusoidal loading. A heavy metal base should be provided for mounting the plunger shaft bearings, and the shaft for the eccentric.

The soil tube is awkward to handle when filling with soil. The loading force tends to consolidate the sand in a vertical direction producing voids at the top of the tube.

Additional access holes would provide a more accurate measurement of wave length. It would be desirable to locate the transducer along the center axis of the tube, with the diaphragm face perpendicular to the axis. This is not easily accomplished, as it is necessary to provide a rigid support for the transducer.

Recording of the wave on a multi-channel recording oscillograph, using several pickups, would allow a more complete analysis of the wave form.

It is felt that the pressure transducer has adequate response and sensitivity for this type of study, however, consideration should be given to mechanical damage to the SR-4 strain gages.

The use of a horizontal tube and horizontal loading is considered desirable as a method for reducing the number of variables involved in the force system. Mounting the tube on a rigid base did not appear to introduce appreciable vertical forces into the system.

The spring loaded plunger appeared to function satisfactorily, however, if the system is used to study wave amplitudes, the force transmitted to the soil would be difficult to determine. Strain in the loading plate plunger and eccentric assembly would have to be accounted for.

B. Measurement of Amplitude

No attempt was made to study amplitude in this investigation, although, the pressure monitor has provision for calibrating the transducer.

If the pressure transducer is statically loaded to a constant value at each access position, the amplitude of the wave will have a direct proportion to the height of the wave on the oscilloscope screen, using a constant vertical gain. Calibration of the transducer will then provide a measure of the amplitude of the wave.

C. Change in Wave Length

Shortening of the wave as it progresses down the tube is shown in Figures 35 - 39. Part of this shortening has been attributed to changes in density. It is possible that the damping force in a visco-elastic medium may contribute to this shortening, and result in a refraction of the wave. The velocity of the wave will be reduced by this refraction. The change in velocity or wave length can then provide a measure of the damping force. The shortening occurs close to the loading plate in an area which can be considered as a transitional zone. The wave, after passing this zone, is less effected by the damping force and acts more nearly as a pure elastic wave.

It is felt that an investigation of this phenomena may prove useful in measuring damping and will contribute to an understanding of the wave forces in the system.

D. Soil Characteristics

The only soil studied in this work was a Port Jefferson Sand, which is a coarse sand altered from its natural state by process screening.

A wide range of soils should be studied under dynamic loading conditions in order to determine the effect of particle size distribution. It might be expected that a sand with a range of particle sizes which would permit complete filling of the voids would not be greatly influenced by damping.

Table 1. *Continued* (see Table 1, p. 10)

Variable	Description	Unit	Mean	SD	Range	Skewness	Kurtosis	Jarque-Bera	Prob	Normality
Age	Age in years	Years	40.1	10.1	18-65	0.00	0.00	0.00	0.99	Yes
Age ²	Age squared	Years ²	1608.0	202.0	324-4225	-0.00	0.00	0.00	0.99	Yes
Age ³	Age cubed	Years ³	64080.0	12000.0	5832-274625	0.00	0.00	0.00	0.99	Yes
Age ⁴	Age to the fourth power	Years ⁴	2568100.0	400000.0	113160-8100625	0.00	0.00	0.00	0.99	Yes
Age ⁵	Age to the fifth power	Years ⁵	102904000.0	16000000.0	6741600-275003125	0.00	0.00	0.00	0.99	Yes
Age ⁶	Age to the sixth power	Years ⁶	4116160000.0	640000000.0	262440000-1771470625	0.00	0.00	0.00	0.99	Yes
Age ⁷	Age to the seventh power	Years ⁷	16466240000.0	2560000000.0	1048576000-1176484375	0.00	0.00	0.00	0.99	Yes
Age ⁸	Age to the eighth power	Years ⁸	65866240000.0	10240000000.0	4177280000-759140625	0.00	0.00	0.00	0.99	Yes
Age ⁹	Age to the ninth power	Years ⁹	263462400000.0	40960000000.0	16778240000-110390625	0.00	0.00	0.00	0.99	Yes
Age ¹⁰	Age to the tenth power	Years ¹⁰	1065862400000.0	163840000000.0	67108800000-1406250000	0.00	0.00	0.00	0.99	Yes
Age ¹¹	Age to the eleventh power	Years ¹¹	4266624000000.0	655360000000.0	268432000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹²	Age to the twelfth power	Years ¹²	17066240000000.0	2621440000000.0	107372800000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹³	Age to the thirteenth power	Years ¹³	68266240000000.0	10485760000000.0	4294400000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹⁴	Age to the fourteenth power	Years ¹⁴	273062400000000.0	41943040000000.0	17160960000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹⁵	Age to the fifteenth power	Years ¹⁵	1092062400000000.0	167774400000000.0	69123840000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹⁶	Age to the sixteenth power	Years ¹⁶	4368062400000000.0	671104000000000.0	27680000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹⁷	Age to the seventeenth power	Years ¹⁷	17472062400000000.0	2684416000000000.0	110502400000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹⁸	Age to the eighteenth power	Years ¹⁸	69888062400000000.0	10737696000000000.0	4418048000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ¹⁹	Age to the nineteenth power	Years ¹⁹	279552062400000000.0	42946304000000000.0	17612160000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²⁰	Age to the twentieth power	Years ²⁰	1118208062400000000.0	171861760000000000.0	70448640000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²¹	Age to the twenty-first power	Years ²¹	4472832062400000000.0	687447040000000000.0	282700800000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²²	Age to the twenty-second power	Years ²²	17891328062400000000.0	2749788800000000000.0	1130803200000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²³	Age to the twenty-third power	Years ²³	71565312062400000000.0	10999104000000000000.0	4519270400000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²⁴	Age to the twenty-fourth power	Years ²⁴	286261248062400000000.0	43996416000000000000.0	18077440000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²⁵	Age to the twenty-fifth power	Years ²⁵	1145044992062400000000.0	175985600000000000000.0	72309760000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²⁶	Age to the twenty-sixth power	Years ²⁶	4580183984062400000000.0	703942400000000000000.0	291240320000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²⁷	Age to the twenty-seventh power	Years ²⁷	18320739904062400000000.0	2815776000000000000000.0	11649600000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²⁸	Age to the twenty-eighth power	Years ²⁸	73283039840624000000000.0	11263040000000000000000.0	46638720000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ²⁹	Age to the twenty-ninth power	Years ²⁹	293132399040624000000000.0	45052160000000000000000.0	186630400000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³⁰	Age to the thirtieth power	Years ³⁰	1172527993606240000000000.0	180208000000000000000000.0	746582400000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³¹	Age to the thirty-first power	Years ³¹	4690111993606240000000000.0	720832000000000000000000.0	2986368000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³²	Age to the thirty-second power	Years ³²	18760447993606240000000000.0	2883328000000000000000000.0	12045440000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³³	Age to the thirty-third power	Years ³³	75041831993606240000000000.0	11533280000000000000000000.0	48181760000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³⁴	Age to the thirty-fourth power	Years ³⁴	299167999936062400000000000.0	46132800000000000000000000.0	192736000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³⁵	Age to the thirty-fifth power	Years ³⁵	1196719999360624000000000000.0	184532800000000000000000000.0	770880000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³⁶	Age to the thirty-sixth power	Years ³⁶	4787271999360624000000000000.0	738132800000000000000000000.0	3087168000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³⁷	Age to the thirty-seventh power	Years ³⁷	19149271999360624000000000000.0	2952528000000000000000000000.0	12348640000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³⁸	Age to the thirty-eighth power	Years ³⁸	76587191999360624000000000000.0	11810240000000000000000000000.0	49384960000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ³⁹	Age to the thirty-ninth power	Years ³⁹	306349591999360624000000000000.0	47240960000000000000000000000.0	197540800000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴⁰	Age to the fortieth power	Years ⁴⁰	1225398391999360624000000000000.0	188963200000000000000000000000.0	790163200000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴¹	Age to the forty-first power	Years ⁴¹	4904033591999360624000000000000.0	755852800000000000000000000000.0	3160672000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴²	Age to the forty-second power	Years ⁴²	19616535919993606240000000000000.0	3023328000000000000000000000000.0	12658880000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴³	Age to the forty-third power	Years ⁴³	78468535919993606240000000000000.0	12093280000000000000000000000000.0	50039040000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴⁴	Age to the forty-fourth power	Years ⁴⁴	313877519993606240000000000000000.0	48373280000000000000000000000000.0	200176000000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴⁵	Age to the forty-fifth power	Years ⁴⁵	125145119993606240000000000000000.0	193502400000000000000000000000000.0	791240000000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴⁶	Age to the forty-sixth power	Years ⁴⁶	500662719993606240000000000000000.0	774022400000000000000000000000000.0	3165696000000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴⁷	Age to the forty-seventh power	Years ⁴⁷	199247919993606240000000000000000.0	313602400000000000000000000000000.0	12622080000000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴⁸	Age to the forty-eighth power	Years ⁴⁸	796983919993606240000000000000000.0	1254422400000000000000000000000000.0	50087040000000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁴⁹	Age to the forty-ninth power	Years ⁴⁹	318873519993606240000000000000000.0	5017624000000000000000000000000000.0	199334400000000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes
Age ⁵⁰	Age to the fiftieth power	Years ⁵⁰	127537919993606240000000000000000.0	20070240000000000000000000000000000.0	796880000000000000000000000000000000-1562500000	0.00	0.00	0.00	0.99	Yes

A fine grained cohesive soil will exhibit dynamic characteristics which depend, to a large extent, on the moisture content. Such a soil, near the liquid limit, should behave as a viscous fluid and at the plastic limit should approximate an elastic solid.

The effect of frequency on a cohesive soil should be greater than on a sand, as the plastic properties depend on the rate and duration of loading. At high frequencies a cohesive soil should behave as an elastic solid.

E. Measuring Technique

The usefulness of this method depends, to a large extent, on the accuracy of the wave length measurements. The wave forms shown in Fig. 34 demonstrate the type of pattern produced on the oscillograph screen. These waves were repetitional in the form shown. Each cycle of loading produced a wave form that duplicated the wave proceeding it and transient waves were a minimum.

The position of the marking pulse was difficult to determine both visually and on the film. This was due to the, sometimes, complex wave pattern which was recorded. The wave forms developed from the data were the result of attempts to fix the position of the marking pulse at a definite value. The position will depend on accuracy with which a sinusoidal wave was produced. This requirement was only approximated and errors are thus introduced in locating the marking pulse.

These errors may, in an individual case, be large, but, observation of a number of wave forms tends to average the wave length out. The

the following conditions are satisfied, then the system (1) is said to be *input-to-state stable* (ISS) [10]:

- (i) For any $\epsilon > 0$, there exist $\delta > 0$ and $\beta \in \mathcal{KL}$ such that for any $\phi \in \mathcal{B}_\delta$ and $w \in \mathcal{B}_\epsilon$, the solution $x(t, t_0, \phi, w)$ of (1) satisfies

$$\|x(t, t_0, \phi, w)\| \leq \beta(\|\phi\|, t - t_0) + \epsilon, \quad \forall t \geq t_0. \quad (2)$$

- (ii) For any $\epsilon > 0$, there exists $\delta > 0$ such that for any $\phi \in \mathcal{B}_\delta$ and $w \in \mathcal{B}_\epsilon$, the solution $x(t, t_0, \phi, w)$ of (1) satisfies

$$\|x(t, t_0, \phi, w)\| \leq \epsilon, \quad \forall t \geq t_0 + T(\|\phi\|, \epsilon), \quad (3)$$

- (iii) For any $\epsilon > 0$, there exists $\delta > 0$ such that for any $\phi \in \mathcal{B}_\delta$ and $w \in \mathcal{B}_\epsilon$, the solution $x(t, t_0, \phi, w)$ of (1) satisfies

$$\|x(t, t_0, \phi, w)\| \leq \epsilon, \quad \forall t \geq t_0. \quad (4)$$

$$T(\|\phi\|, \epsilon) = \inf_{\tau \geq 0} \beta(\|\phi\|, \tau) \leq \epsilon.$$

- (iv) For any $\epsilon > 0$, there exists $\delta > 0$ such that for any $\phi \in \mathcal{B}_\delta$ and $w \in \mathcal{B}_\epsilon$, the solution $x(t, t_0, \phi, w)$ of (1) satisfies

$$\|x(t, t_0, \phi, w)\| \leq \epsilon, \quad \forall t \geq t_0 + T(\|\phi\|, \epsilon), \quad (5)$$

- (v) For any $\epsilon > 0$, there exists $\delta > 0$ such that for any $\phi \in \mathcal{B}_\delta$ and $w \in \mathcal{B}_\epsilon$, the solution $x(t, t_0, \phi, w)$ of (1) satisfies

$$\|x(t, t_0, \phi, w)\| \leq \epsilon, \quad \forall t \geq t_0 + T(\|\phi\|, \epsilon), \quad (6)$$

$$T(\|\phi\|, \epsilon) = \inf_{\tau \geq 0} \beta(\|\phi\|, \tau) \leq \epsilon.$$

$$T(\|\phi\|, \epsilon) = \inf_{\tau \geq 0} \beta(\|\phi\|, \tau) \leq \epsilon.$$

$$T(\|\phi\|, \epsilon) = \inf_{\tau \geq 0} \beta(\|\phi\|, \tau) \leq \epsilon.$$

- (vi) For any $\epsilon > 0$, there exists $\delta > 0$ such that for any $\phi \in \mathcal{B}_\delta$ and $w \in \mathcal{B}_\epsilon$, the solution $x(t, t_0, \phi, w)$ of (1) satisfies

$$\|x(t, t_0, \phi, w)\| \leq \epsilon, \quad \forall t \geq t_0 + T(\|\phi\|, \epsilon), \quad (7)$$

$$T(\|\phi\|, \epsilon) = \inf_{\tau \geq 0} \beta(\|\phi\|, \tau) \leq \epsilon.$$

$$T(\|\phi\|, \epsilon) = \inf_{\tau \geq 0} \beta(\|\phi\|, \tau) \leq \epsilon.$$

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$$T(\|\phi\|, \epsilon) = \inf_{\tau \geq 0} \beta(\|\phi\|, \tau) \leq \epsilon.$$

fact that the final value was reproducible within narrow limits is demonstrated by the width of the shaded area of Fig. 40 and Fig. 41.

The technique can be much improved by elimination of some of the weak features of the equipment as discussed in Part VI - A.

PART VII

CONCLUSIONS AND RECOMMENDATIONS

1. The pressure transducer, utilizing SR-4 Strain Gages, provides a convenient method for measuring wave pressure. Faithful recording of wave pressure is dependent on a rigid support for the diaphragm. In studying soil pressures, this becomes a prime factor for consideration because of changing pressure on the diaphragm as a result of support movement. For this reason, a pressure diaphragm which is buried in the soil must be supported so that support movement will not generate pressure.

2. The method used for wave form analysis in this study can produce reliable wave length and velocity data even though the wave form is other than a pure sinusoidal wave. Accurate determination of wave length will depend upon establishing definite locations of peak points on the wave form.

3. Wave length changes as the wave progresses down the tube. The reason for this change in wave length will require further investigation. Both changes in density and damping appear to offer possible answers.

4. The wave lengths measured in this study differ considerably from transverse surface waves measured by others.^{2,14,23} The range of wave lengths shown in Fig. 40 varies from 23 to 35 inches. As the frequency increases the wave length decreases to an asymptotic value of approximately 20 inches. Increasing frequency beyond about 40 cycles per second should produce only minor decreases in wave length.

5. Propagation velocities in soil are not constant for a particular type of soil and independent of frequency. A comparison of properties of soil is possible when frequency is maintained constant. It should be advantageous to study velocity frequency relationships above 30 cycle per second. A correlation should obtain between resonant frequency and slope of the velocity - forcing frequency curve.

6. The type of waves recorded by the pressure transducer, when placed in the access holes of the tube, is unknown. The extremely low velocities encountered in this investigation give rise to the belief that tube geometry must be considered in the wave analysis.

Waves reflected from the end-block may have sufficient amplitude to produce a standing wave pattern, in which case, the marking pulse will define a half wave length between points C_1 and C_2 of equation (21). The actual distance between the two points, in terms of a single wave travelling down the tube, would then be a phase lag of 90° or one quarter of the wave length of the forcing wave.

This phenomena can be investigated by measuring amplitudes of the wave form along the length of the tube. A standing wave is characterized by a pivot point in the wave form at which pressure change with respect to time is a minimum.

7. All of the phenomena associated with this apparatus are not easily separated. It is difficult to analyze the type and source of all of the wave forces which may contribute to the wave form which is recorded. Further study of waves generated in soil, when contained in a tube, will be necessary before a satisfactory evaluation can be made of the apparatus developed in this investigation.

PART VIII

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PART IX
APPENDIX
Data Sheet

Run No.	n	Access Position	Sweep Freqs	Vert Gain	Horiz Gain	Position of Marking Pulse
1	14.5	1	10	1.4	.1	4
		2	6	1.20	1.2	1.5
		3	10	1.25	1.25	7
		4	10	1.5	1.0	9.5
		5	11	1.3	.1	5
		6	9	1.2	.11	4.5
2	21.75	6	10	1.2	.11	3.5
		5	11	1.25	.11	8.5
		4	11	1.25	.12	
		3	5	1.25	.14	7.5
		2	10	1.5	.12	5
		1	10	1.2	.12	3
3	29.0	1	10	1.2	.12	6
		2	10	1.2	.12	1
		3	10	1.2	.12	5
		4	10	1.2	.12	1.5
		5	10	1.2	.12	4
		6	10	1.2	.12	4.5
4	21.75	6	7	1.5	.12	3.5
		5	7	1.1	.12	8
		4	7	1.1	.12	5
		3	6	1.1	.12	1
		2	6	1.1	.12	6.5
		1	6	1.25	.12	1.5
5	14.5	1	10	1.25	.12	5.5
		2	10	1.25	.12	4.5
		3	10	1.25	.12	3
		4	12	1.25	.12	2.5
		5	10	1.25	.12	3.5
		6	10	1.25	.12	1.5



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